The Proof of the Position is in the Testing

Reliable Accuracy Determination for Spatial Data

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Presentation abstract. There is tremendous confusion on the issue of spatial accuracy. A number of questions must be answered for an accuracy assessment to be meaningful. Do the values reflect "accuracy" (closeness to "truth") or "precision" (repeatability)? What confidence level should be used (95%, 90%, 68%, and 39% are common values). How do you compute accuracy? Where does RMSE fit in with all this? Are horizontal and vertical accuracies handled in the same way? Are the internal accuracy estimates of GPS software reliable? Should you worry about trivial GPS vectors in survey control networks? Are standards available to provide guidance?

Computing error circle and ellipse from standard error components

Accuracies are often given in terms of *standard errors* in the north, east, and up (*y*, *x*, and *z*) components. Frequently these values are given as Root Mean Square Error (RMSE), which, for the purpose of this handout can be considered equivalent to the standard error.

The north and east (*y* and *x*) components can be converted to a horizontal (circular) accuracy consistent with the approach used by the National Standard for Spatial Data Accuracy (NSSDA) as developed by the Federal Geographic Data Committee (1998, Part 3). Error ellipse axes and rotation can also be computed from the north and east standard error components and horizontal covariance, as illustrated below.

Equation 1. Horizontal (circular) accuracy computed from north and east accuracy components (at the 95% confidence level per NSSDA)

$$CEP_{95} = 2.4477\,\frac{\sigma_N + \sigma_E}{2}$$

where CEP_{95} is the estimated Circular Error Probable (horizontal accuracy) at 95% confidence

σ_N and σ_E are the north (*y*) and east (*x*) standard errors, respectively (equivalent to the horizontal RMSE components)

The value 2.4477 is the *bivariate* scalar for a confidence level of 95% (see Table 1 below for this scalar at this and other confidence levels). Note that *CEP* is often computed at the 50% confidence level (see Table 1 for 50% confidence level error scalar).

Equation 2. Vertical accuracy computed from vertical standard error (at the 95% confidence level per NSSDA)

$$E_{95}^{U} = 1.9600\ \sigma_U$$

where E_{95}^{U} is the estimated vertical accuracy at 95% confidence

σ_U is the "up" (z) standard error (equivalent to the vertical RMSE)

The value 1.9600 is the *univariate* scalar for a confidence level of 95% (as shown Table 1 below).

Table 1. Values used to scale standard errors (accuracies) to various confidence levels. The *univariate scalar* is used for single error components, such as vertical error. The *bivariate scalar* is used for dual (two-dimensional) error components, such as horizontal error, and can be used to scale an error ellipse to a desired confidence level. The *trivariate scalar* is rarely used but is provided here for the sake of completeness. It is for three-dimensional error components and can be used for scaling an error ellipsoid to a desired confidence level. In all cases, these scalars are based on the normal probability distribution of random variables, and the multivariate scalars are for jointly distributed random variables.

Univariate scalars		**Bivariate scalars**		**Trivariate scalars**	
Scalar	**Confidence level**	**Scalar**	**Confidence level**	**Scalar**	**Confidence level**
0.6745	50.00%	1.0000	39.35%	1.0000	19.87%
1.0000	68.27%	1.1774	50.00%	1.5382	50.00%
1.6449	90.00%	2.0000	86.47%	2.0000	73.85%
1.9600	**95.00%**	2.1460	90.00%	2.5003	90.00%
2.0000	95.45%	**2.4477**	**95.00%**	**2.7955**	**95.00%**
2.5758	99.00%	3.0000	98.89%	3.0000	97.07%
3.0000	99.73%	3.0349	99.00%	3.3682	99.00%
3.2905	99.90%	3.7169	99.90%	4.0331	99.90%

Equation 3. Horizontal covariance and correlation

$$\sigma_{NE} = \rho\,\sigma_N\,\sigma_E$$

where σ_{NE} is the horizontal covariance

ρ is the horizontal correlation

The horizontal covariance can be computed directly from the delta north and delta east values used for computing the NSSDA north and east errors (i.e., the *y* and *x* RMSE values). This is done by summing the product of the delta north × delta east and dividing this sum by the total number of summed values.

Equation 4. Horizontal error ellipse axes computed from standard errors and covariance (at 95% confidence)

$$a,\ b\ =\ 2.4477\sqrt{\frac{1}{2}\left[\sigma_N^2+\sigma_E^2\ \pm\ \sqrt{\left(\sigma_N^2-\sigma_E^2\right)^2+4\sigma_{NE}^2}\right]}$$

where *a* and *b* are the error ellipse semi-major and semi-minor axes, scaled to 95% confidence (note that the "±" operator allows computation of both *a* and *b* with this one equation, and that *a* is always greater than *b*).

Equation 5. Horizontal error ellipse rotation computed from standard errors and covariance (standard error values are given in the NGS RDF file)

$$\theta=\frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{NE}}{\sigma_E^2-\sigma_N^2}\right)$$

where θ is the rotation angle of the semi-major axis, with respect to the east direction (positive counterclockwise). If $\sigma_N > \sigma_E$, rotation is with respect to the *positive* east axis. If $\sigma_N < \sigma_E$, rotation is with respect to the *negative* east axis. If $\sigma_N = \sigma_E$, then θ = ±45°, where the sign of the rotation is determined by the sign of σ_{NE}.

Equation 6. Three-dimensional (spherical) accuracy computed from north, east, and up accuracy components (at the 95% confidence level)

$$SEP_{95} = 2.7955\,\frac{\sigma_N+\sigma_E+\sigma_U}{3}$$

where SEP_{95} is the estimated Spherical Error Probable (three-dimensional accuracy) at 95% confidence, and all other variables are as defined previously. Note that spherical accuracy estimation is not included in the NSSDA, and is provided here for the sake of completeness. The value 2.7955 is the *trivariate* scalar for a confidence level of 95% (as shown in Table 1). As with *CEP*, *SEP* is often computed at 50% confidence.

SPATIAL DATA ACCURACY STANDARDS, SPECIFICATIONS, AND GUIDELINES

American Land Title Association, American Congress on Surveying & Mapping, and National Society of Professional Surveyors (2005) *2005 Minimum Standard Detail Requirements for ALTA/ACSM Land Title Surveys*, 6 pp., http://www.acsm.net/alta.html.

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Bureau of the Budget (1947) *National Map Accuracy Standards*, Office of Management and Budget, Washington, D.C., 1 p. http://rockyweb.cr.usgs.gov/nmpstds/acrodocs/ nmas/NMAS647.PDF . ***[Note: These standards have been superseded by the FGDC 1998 standards and are NOT recommended for use]***

Federal Emergency Management Agency (2005) *Guidelines and Specifications for Flood Hazard Mapping Partners*, FEMA Map Modernization Program, April 2003 version. Consists of 3 volumes (337 pp.), 13 appendices (1207 pp.), and 5 supporting documents (85 pp.), for a total of 1629 pp., http://www.fema.gov/library/viewRecord.do?id=2206.

Federal Geographic Data Committee (1998) *Geospatial Positioning Accuracy Standards*, FGDC-STD-007.2-1998, Federal Geographic Data Committee, Reston, Virginia, USA, 128 pp., http://www.fgdc.gov/standards/projects/FGDC-standards-projects/accuracy/, [includes Standards for Geodetic Networks (Part 2), National Standard for Spatial Data Accuracy (Part 3), and Standards for Architecture, Engineering, Construction (A/E/C) and Facility Management (Part 4)].

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Zilkoski, D.B., D'Onofrio, J. D. and Frakes, S. J. (1997) Guidelines for Establishing GPS-Derived Ellipsoid Heights (Standards: 2 cm and 5 cm), version 4.3, *NOAA Technical Memorandum NOS NGS-58*, National Geodetic Survey, Silver Springs, MD, USA, 22 pp., http://www.ngs.noaa.gov/PUBS_ LIB/NGS-58.pdf.

